

# 1 Performance tests of poisson equation solvers

## 1.1 Problem description

Poisson equation:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = b$$

Area:  $\Omega = (0, L_x) \times (0, L_y)$ ,  $L_x = 1$ ,  $L_y = 1$

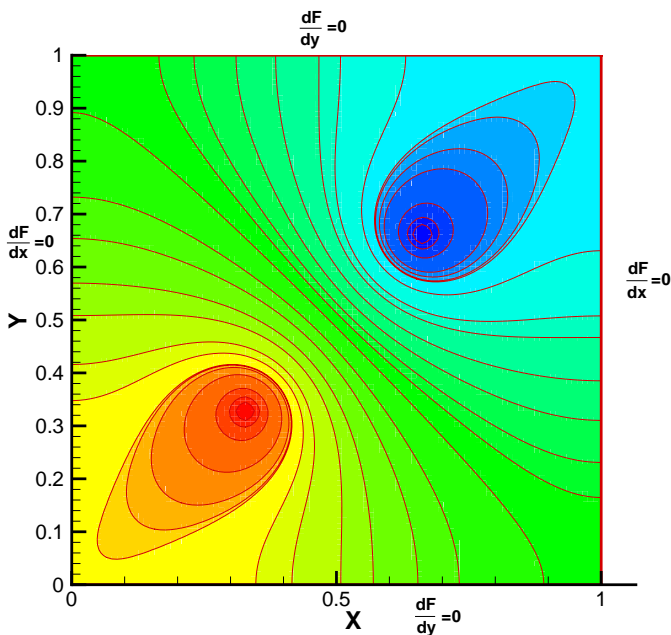
Boundary conditions:  $\frac{\partial F}{\partial n} = 0$  everywhere on the boundary

Sources:

$$\int_{\Omega} b = 0$$

$$b(L_x/3, L_y/3) = 1, \quad b(2L_x/3, 2L_y/3) = -1$$

Mesh: Uniform,  $N_x = N_y$ .



## 1.2 Solvers

- Gauss-Seidel + TDMA iterative solver (GS)

Accuracy of iterative process:

$$\frac{\|Ax^h - b\|}{\|b\|} < 10^{-8}$$

- Fourier solver DFT + TDMA (DFT)

Decomposition of 2D problem using DFT into  $N_x$  1D problems with tridiagonal systems solved with TDMA.

Restrictions on X axis:

Boundary conditions:

1) Fixed:  $F(0) = F(L_x) = 0$

2) Free:  $\frac{\partial F}{\partial x} = 0$ ,  $x = 0$ ,  $\frac{\partial F}{\partial x} = 0$ ,  $x = L_x$

3) Periodic

Mesh: Uniform step on X axis.  $N_x = 2^a$  (or  $N_x = 2^a + 1$  for free boundary conditions)

Accuracy:

$$\frac{\|Ax^h - b\|}{\|b\|} < 10^{-13}$$

- Fourier solver FFT + TDMA (FFT)  
Decomposition of 2D problem using FFT into Nx 1D problems with tridiagonal systems solved with TDMA.

Restrictions are the same as in previous case.

Accuracy:

$$\frac{\|Ax^h - b\|}{\|b\|} < 10^{-13}$$

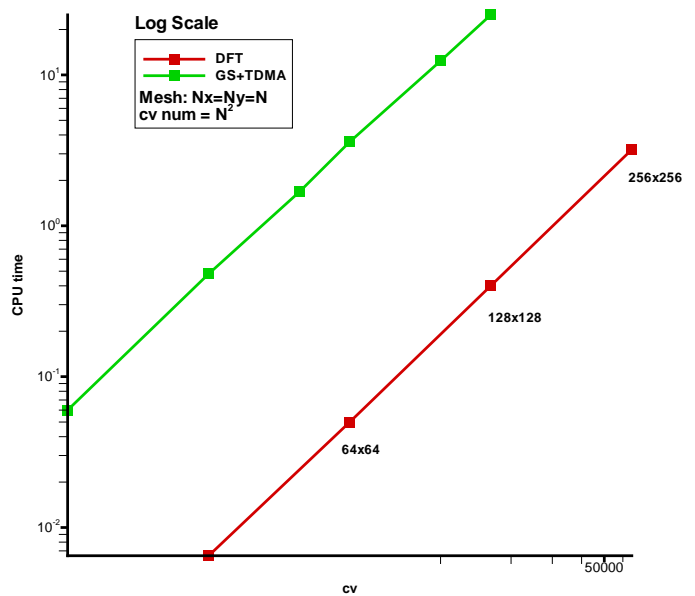
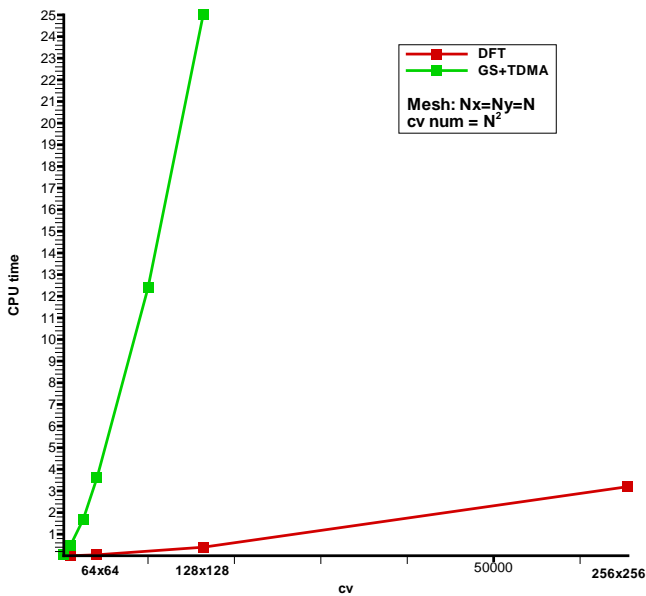
- Fourier solver FFT + FFT (FFTx2) Decomposition of 2D problem using FFT into Nx 1D problems with tridiagonal systems solved with FFT.

All restrictions from previous cases applied to both axes X and Y.

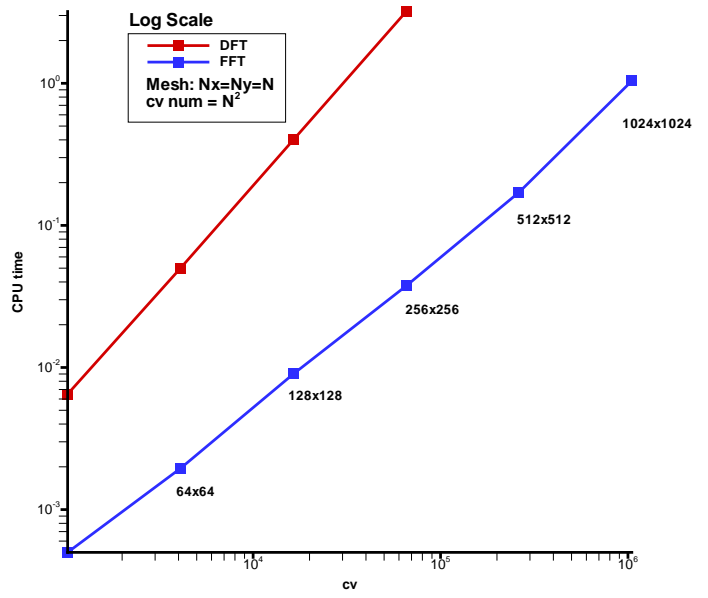
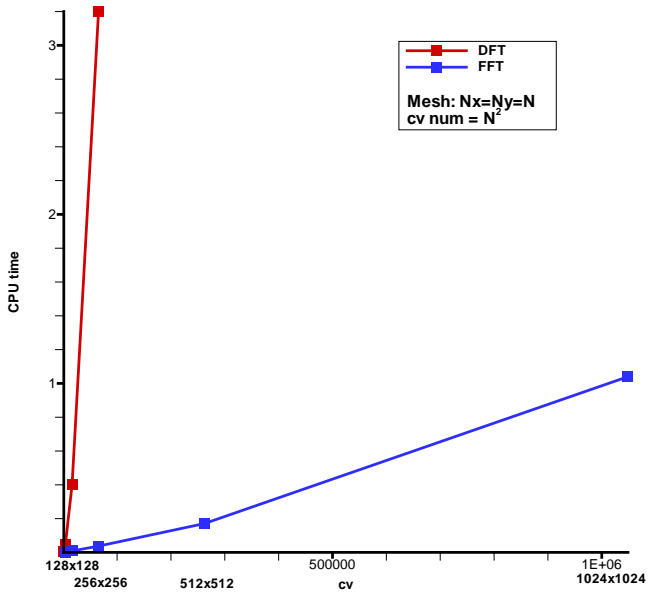
Accuracy:

$$\frac{\|Ax^h - b\|}{\|b\|} < 10^{-14}$$

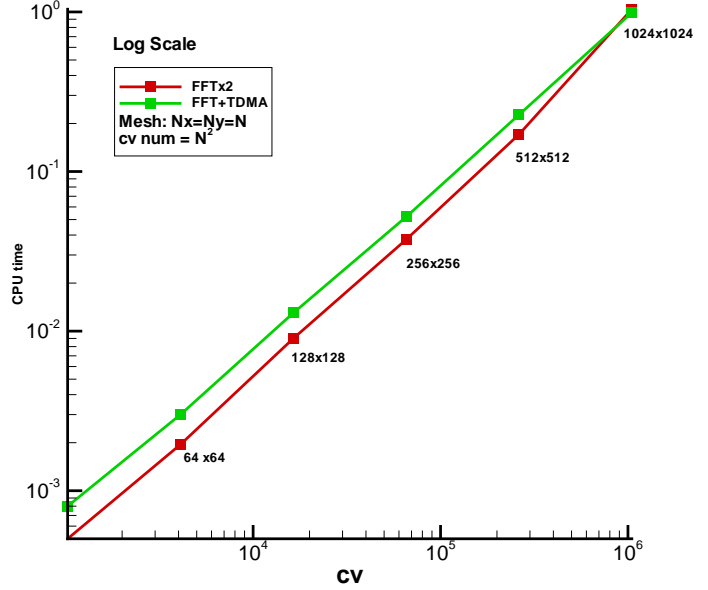
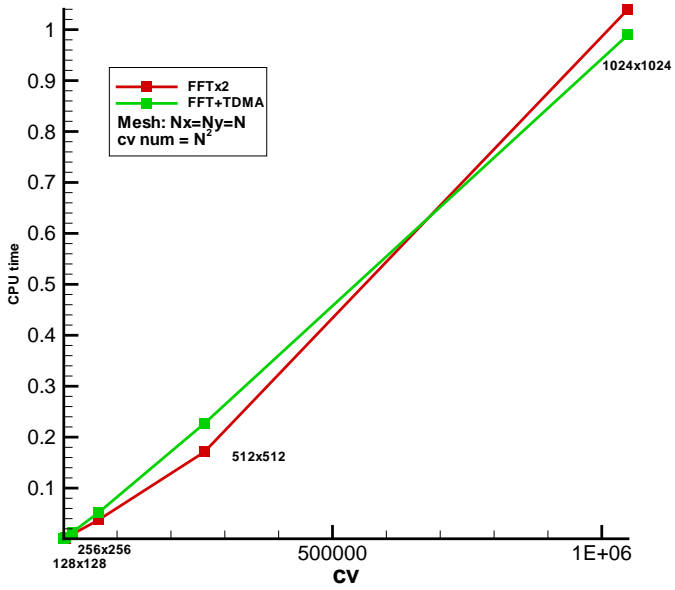
### 1.3 GS vs DFT



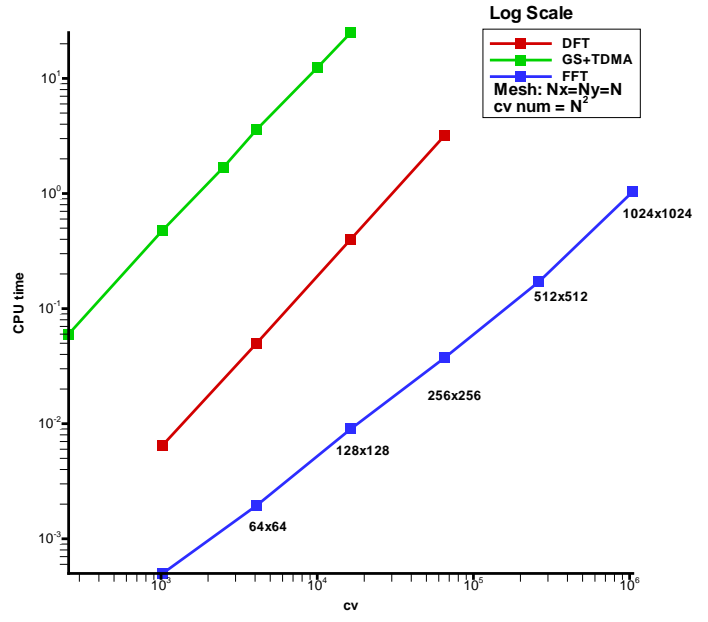
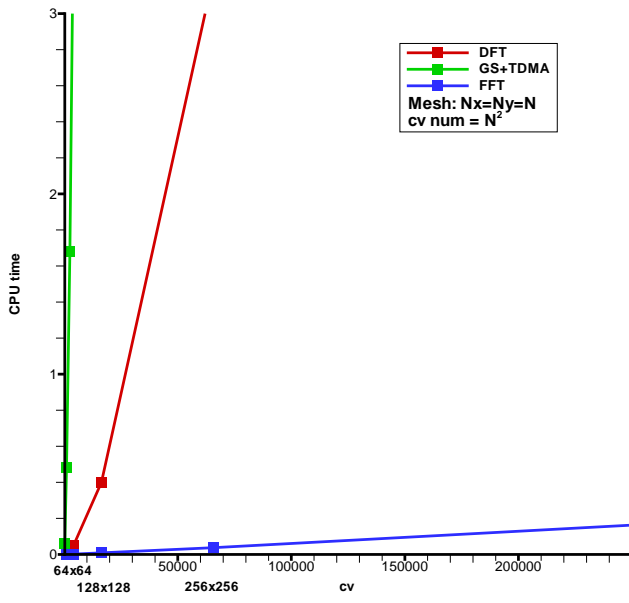
### 1.4 DFT vs FFT



### 1.5 FFT vs FFTx2



## 1.6 GS, DFT and FFT comparison



## REFERENCES

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