# Step5

## 1 Hight order schemes: QUICK, SMART.

### 1.1 Diagonal flow

**Velocity field:**
$u(x,y) = U_0\ cos(\alpha)\quad v(x,y) = V_0\ sin(\alpha)$

The boundary conditions are:
$\phi(0,y) = \phi_1,\quad 0 \le y \le L_y\quad \phi(x,M) = \phi_1,\quad 0 \le x \le L_x$
$\phi(L,y) = \phi_2,\quad 0 \le y \le L_y\quad \phi(x,0) = \phi_2,\quad 0 \le x \le L_x$
Source: S=0

**Exact solution**
For $Pe = \infty$:
$\phi = \phi_1$ above the diagonal, $\phi = \phi_2$ above the below

**Numerical results**
Pe=$10^8$: $L_x = L_y = 1,\ \rho = 10000,\ U_0 = V_0 = 1,\ \mu = 0.0001,\ \lambda = 1$

Table 1: Error in norm $l_2$ for different schemes and meshes.

| UPWIND | QUICK | SMART | Num of cv |
|---|---|---|---|
| 0.255 | 0.238 | 0.238 | 100cv (10x10) |
| 0.215 | 0.174 | 0.174 | 400cv (20x20) |
| 0.182 | 0.127 | 0.127 | 1600cv (40x40) |
| 0.154 | 0.095 | 0.095 | 6400cv (80x80) |
| 0.131 | 0.074 | 0.074 | 25600cv (160x160) |

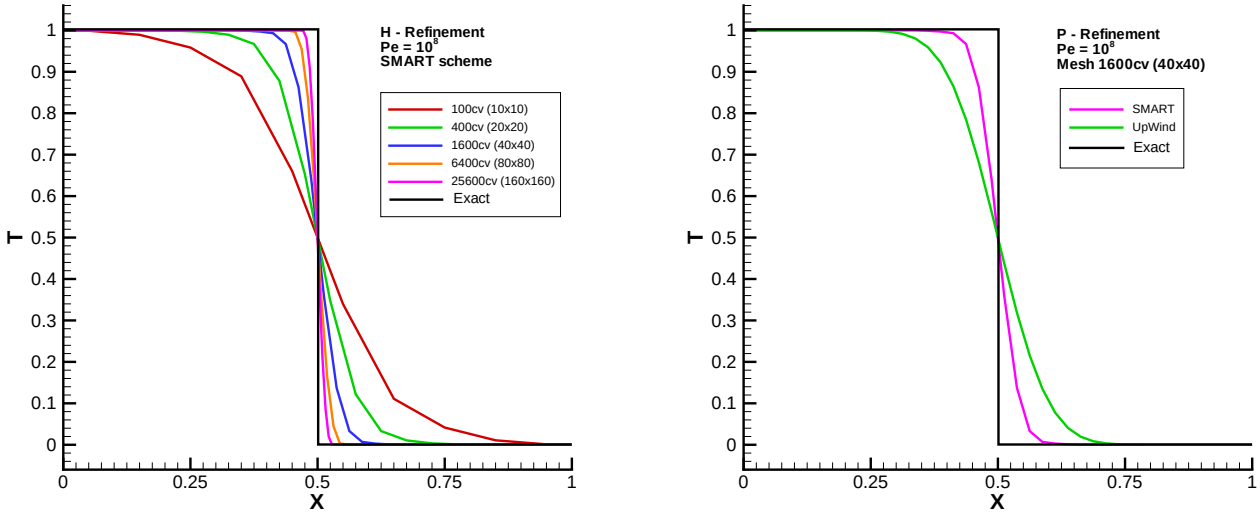


Figure 1: H-refinement for SMART scheme (left) and comparison of SMART and Upwind (right). Values on line $x = L_y - y$

Comments:
Results of QUICK and SMART schemes are equal. Upwind gives distinctly less accurate result.

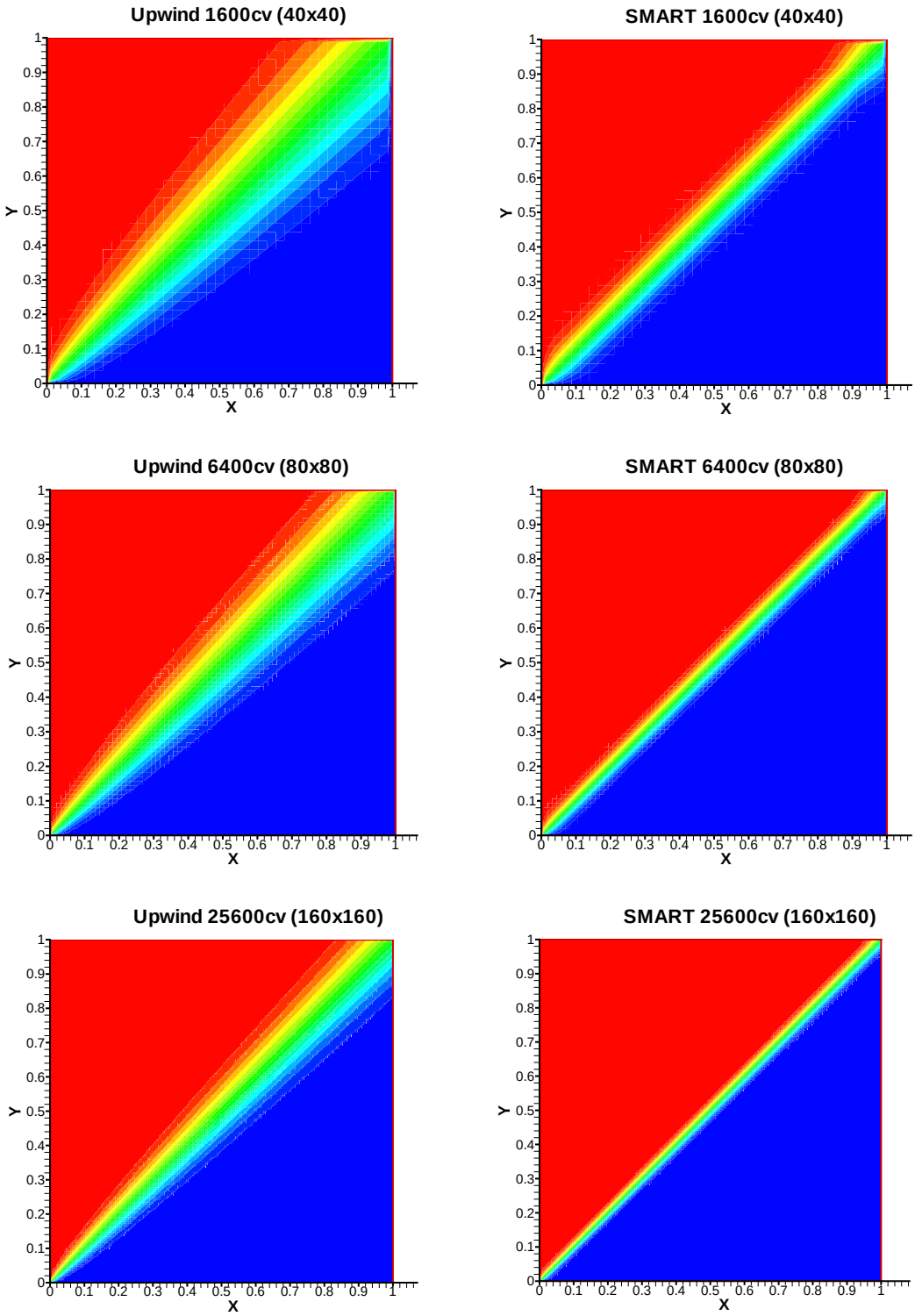


Figure 2: 2D results of SMART and Upwind schemes for different meshes.

Comments:
This comparison of 2D results shows explicitly that upwind scheme has much bigger false diffusion. So upwind scheme gives greatly less accurate solution than SMART scheme.

# 2 Driven Cavity Problem

Boundary conditions:
$u = 0,\ v = 0,\ T = 0,\quad y = 0,\ 0 \le x \le L_x$
$u = u_0,\ v = 0,\ T = 0,\quad y = L_y,\ 0 \le x \le L_x$
$u = 0,\ v = 0,\ T = 0,\quad x = 0,\ 0 \le y \le L_y$
$u = 0,\ v = 0,\ T = 0,\quad x = L_x,\ 0 \le y \le L_y$

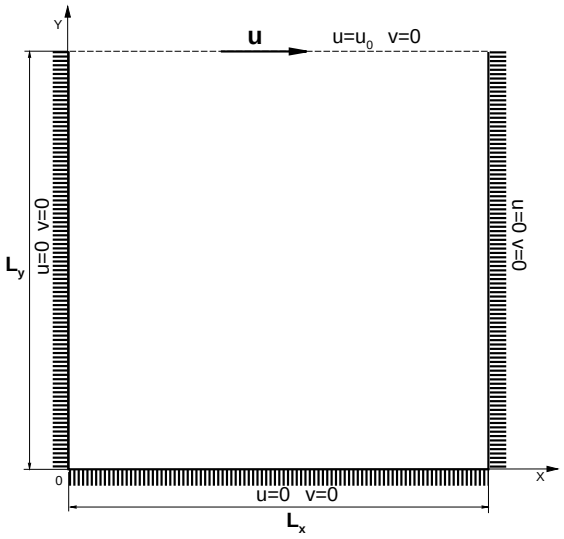


Figure 3: Driven cavity problem scketch

## 2.1 Numerical results

Problem parameters:
Re=1000
$L_x = L_y = 1 \quad u0 = 1 \quad \rho = 1 \quad \mu = 0.01$
$\beta = 0 \quad c_p = 1 \quad k = 1 \quad g = 10$

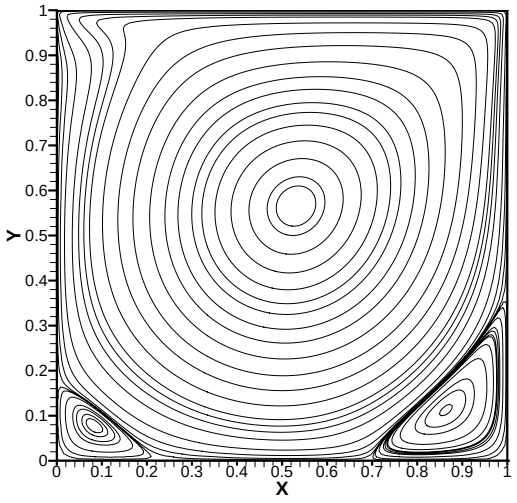


Figure 4: Streamline contours. Re=1000. Mesh 25600cv (160x160), SMART scheme.

**H-refinement and comparison of SMART and Upwind schemes for Re=1000**

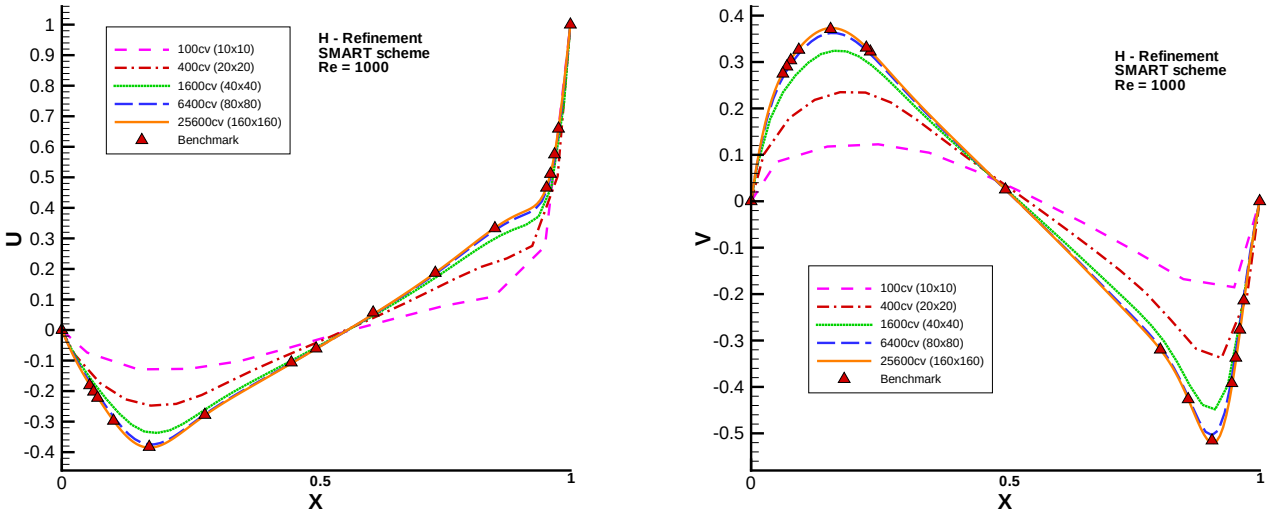


Figure 5: H-refinement results

Table 2: U-velocity. Results of SMART and upwind schemes (Re=1000)

| Y | Bench. | 25600cv (160x160) | | 6400cv (80x80) | | 1600cv (40x40) | | 400cv (20x20) | | 100cv (10x10) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | SMART | UDS | SMART | UDS | SMART | UDS | SMART | UDS | SMART | UDS |
| 0.0000 | 0.0000 | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) |
| 0.0547 | -0.1811 | -0.1787 ( -0.0024) | -0.1784 ( -0.0027) | -0.1702 ( -0.0109) | -0.1601 ( -0.0210) | -0.1550 ( -0.0261) | -0.1199 ( -0.0612) | -0.1354 ( -0.0457) | -0.0850 ( -0.0961) | -0.0767 ( -0.1044) | -0.0686 ( -0.1125) |
| 0.0625 | -0.202 | -0.1995 ( -0.0025) | -0.1955 ( -0.0065) | -0.1901 ( -0.0119) | -0.1746 ( -0.0274) | -0.1734 ( -0.0286) | -0.1326 ( -0.0694) | -0.1501 ( -0.0519) | -0.0941 ( -0.1079) | -0.0809 ( -0.1211) | -0.0708 ( -0.1312) |
| 0.0703 | -0.2222 | -0.2199 ( -0.0023) | -0.2112 ( -0.0110) | -0.2098 ( -0.0124) | -0.1881 ( -0.0341) | -0.1898 ( -0.0324) | -0.1415 ( -0.0807) | -0.1647 ( -0.0575) | -0.1032 ( -0.1190) | -0.0852 ( -0.1370) | -0.0731 ( -0.1491) |
| 0.1016 | -0.2973 | -0.2966 ( -0.0007) | -0.2633 ( -0.0340) | -0.2838 ( -0.0135) | -0.2296 ( -0.0677) | -0.2524 ( -0.0449) | -0.1727 ( -0.1246) | -0.2019 ( -0.0954) | -0.1237 ( -0.1736) | -0.1024 ( -0.1949) | -0.0822 ( -0.2151) |
| 0.1719 | -0.3829 | -0.3853 ( 0.0024) | -0.3221 ( -0.0608) | -0.3743 ( -0.0086) | -0.2767 ( -0.1062) | -0.3344 ( -0.0485) | -0.2095 ( -0.1734) | -0.2463 ( -0.1366) | -0.1488 ( -0.2341) | -0.1285 ( -0.2544) | -0.0955 ( -0.2874) |
| 0.2813 | -0.2781 | -0.2794 ( 0.0013) | -0.2512 ( -0.0268) | -0.2763 ( -0.0018) | -0.2347 ( -0.0434) | -0.2612 ( -0.0169) | -0.2017 ( -0.0764) | -0.2079 ( -0.0702) | -0.1456 ( -0.1325) | -0.1193 ( -0.1588) | -0.0876 ( -0.1904) |
| 0.4513 | -0.1064 | -0.1094 ( 0.0030) | -0.0924 ( -0.0140) | -0.1080 ( 0.0016) | -0.0849 ( -0.0215) | -0.1006 ( -0.0058) | -0.0860 ( -0.0204) | -0.0765 ( -0.0299) | -0.0757 ( -0.0307) | -0.0562 ( -0.0502) | -0.0489 ( -0.0575) |
| 0.5 | -0.0608 | -0.0617 ( 0.0009) | -0.0523 ( -0.0085) | -0.0612 ( 0.0004) | -0.0482 ( -0.0126) | -0.0570 ( -0.0038) | -0.0498 ( -0.0110) | -0.0420 ( -0.0188) | -0.0473 ( -0.0135) | -0.0328 ( -0.0280) | -0.0331 ( -0.0277) |
| 0.6122 | 0.057 | 0.0516 ( 0.0054) | 0.0441 ( 0.0129) | 0.0502 ( 0.0068) | 0.0390 ( 0.0180) | 0.0466 ( 0.0104) | 0.0308 ( 0.0262) | 0.0394 ( 0.0176) | 0.0181 ( 0.0389) | 0.0187 ( 0.0383) | 0.0030 ( 0.0540) |
| 0.7344 | 0.187 | 0.1876 ( -0.0006) | 0.1617 ( 0.0253) | 0.1840 ( 0.0030) | 0.1431 ( 0.0439) | 0.1713 ( 0.0157) | 0.1145 ( 0.0725) | 0.1360 ( 0.0510) | 0.0774 ( 0.1096) | 0.0718 ( 0.1152) | 0.0383 ( 0.1487) |
| 0.8516 | 0.333 | 0.3346 ( -0.0016) | 0.2769 ( 0.0561) | 0.3269 ( 0.0061) | 0.2364 ( 0.0966) | 0.2992 ( 0.0338) | 0.1831 ( 0.1499) | 0.2215 ( 0.1115) | 0.1287 ( 0.2043) | 0.1124 ( 0.2206) | 0.0841 ( 0.2489) |
| 0.9531 | 0.466 | 0.4670 ( -0.0010) | 0.4235 ( 0.0425) | 0.4567 ( 0.0093) | 0.3951 ( 0.0709) | 0.4315 ( 0.0345) | 0.3651 ( 0.1009) | 0.4039 ( 0.0621) | 0.3583 ( 0.1077) | 0.3190 ( 0.1470) | 0.3055 ( 0.1605) |
| 0.9609 | 0.5112 | 0.5122 ( -0.0011) | 0.4746 ( 0.0366) | 0.5021 ( 0.0091) | 0.4494 ( 0.0618) | 0.4616 ( 0.0496) | 0.4064 ( 0.1048) | 0.4396 ( 0.0716) | 0.4001 ( 0.1110) | 0.4323 ( 0.0789) | 0.4210 ( 0.0902) |
| 0.9688 | 0.5749 | 0.5765 ( -0.0016) | 0.5445 ( 0.0304) | 0.5589 ( 0.0160) | 0.5150 ( 0.0599) | 0.5375 ( 0.0374) | 0.4932 ( 0.0817) | 0.4757 ( 0.0992) | 0.4426 ( 0.1323) | 0.5470 ( 0.0279) | 0.5380 ( 0.0369) |
| 0.9766 | 0.6592 | 0.6599 ( -0.0007) | 0.6335 ( 0.0257) | 0.6491 ( 0.0101) | 0.6135 ( 0.0457) | 0.6238 ( 0.0354) | 0.5903 ( 0.0689) | 0.5358 ( 0.1234) | 0.5094 ( 0.1498) | 0.6603 ( -0.0011) | 0.6535 ( 0.0057) |
| 1.0000 | 1.0000 | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) |

Table 3: V-velocity. Results of SMART and upwind schemes (Re=1000)

| X | Bench. | 25600cv (160x160) SMART | 25600cv (160x160) UDS | 6400cv (80x80) SMART | 6400cv (80x80) UDS | 1600cv (40x40) SMART | 1600cv (40x40) UDS | 400cv (20x20) SMART | 400cv (20x20) UDS | 100cv (10x10) SMART | 100cv (10x10) UDS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0000 | 1.0000 | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) | 1.0000 (0.0000) |
| 0.9688 | -0.2138 | -0.2191 ( 0.0053) | -0.2141 ( 0.0003) | -0.2045 ( -0.0093) | -0.1971 ( -0.0167)) | -0.2071 ( -0.0067) | -0.1652 ( -0.0486) | -0.2217 ( 0.0079) | -0.1902 ( -0.0236) | -0.1156 ( -0.0982) | -0.1124 ( -0.1014) |
| 0.9609 | -0.2767 | -0.2835 ( 0.0068) | -0.2716 ( -0.0051) | -0.2664 ( -0.0103) | -0.2468 ( -0.0299) | -0.2609 ( -0.0157) | -0.2075 ( -0.0692) | -0.2426 ( -0.0341) | -0.2023 ( -0.0743) | -0.1448 ( -0.1319) | -0.1408 ( -0.1359) |
| 0.9531 | -0.3371 | -0.3447 ( 0.0075) | -0.3242 ( -0.0130) | -0.3248 ( -0.0123) | -0.2921 ( -0.0450) | -0.3060 ( -0.0311) | -0.2369 ( -0.1002) | -0.2633 ( -0.0739) | -0.2143 ( -0.1228 | -0.1737 ( -0.1634) | -0.1689 ( -0.1682) |
| 0.9453 | -0.3919 | -0.3991 ( 0.0072) | -0.3693 ( -0.0226) | -0.3794 ( -0.0125) | -0.3318 ( -0.0600) | -0.3511 ( -0.0408) | -0.2664 ( -0.1255) | -0.2840 ( -0.1079) | -0.2263 ( -0.1656) | -0.1844 ( -0.2075) | -0.1781 ( -0.2138) |
| 0.9063 | -0.5155 | -0.5199 ( 0.0044) | -0.4691 ( -0.0464) | -0.5038 ( -0.0117) | -0.4217 ( -0.0938) | -0.4460 ( -0.0695) | -0.3376 ( -0.1779) | -0.3303 ( -0.1852) | -0.2511 ( -0.2644) | -0.1776 ( -0.3379) | -0.162 ( -0.3534) |
| 0.8594 | -0.4266 | -0.4262 ( -0.0005) | -0.3934 ( -0.0332) | -0.4216 ( -0.0051) | -0.3683 ( -0.0584) | -0.3903 ( -0.0364) | -0.3160 ( -0.1107) | -0.2983 ( -0.1284) | -0.2265 ( -0.2002) | -0.1694 ( -0.2573) | -0.1428 ( -0.2838) |
| 0.8047 | -0.3197 | -0.3194 ( -0.0003) | -0.2801 ( -0.0396) | -0.3148 ( -0.0049) | -0.2621 ( -0.0576) | -0.2918 ( -0.0278) | -0.2403 ( -0.0793) | -0.2312 ( -0.0884) | -0.1749 ( -0.1448) | -0.1390 ( -0.1806) | -0.1093 ( -0.2104) |
| 0.5 | 0.02526 | 0.0254 ( -0.0001) | 0.0303 ( -0.0051) | 0.0249 ( 0.0004) | 0.0336 ( -0.0084) | 0.0268 ( -0.0015) | 0.0439 ( -0.0186) | 0.0347 ( -0.0095) | 0.0559 ( -0.0307) | 0.0360 ( -0.0107) | 0.0416 ( -0.0164) |
| 0.2344 | 0.3224 | 0.3232 ( -0.0009) | 0.2869 ( 0.0354) | 0.3170 ( 0.0053) | 0.2546 ( 0.0678) | 0.2950 ( 0.0273) | 0.2024 ( 0.1199) | 0.2301 ( 0.0922) | 0.1421 ( 0.1802) | 0.1219 ( 0.2004) | 0.0899 ( 0.2325) |
| 0.2266 | 0.3307 | 0.3317 ( -0.0010) | 0.2920 ( 0.0387) | 0.3251 ( 0.0057) | 0.2577 ( 0.0731) | 0.3010 ( 0.0297) | 0.2035 ( 0.1272) | 0.2335 ( 0.0973) | 0.1429 ( 0.1879) | 0.1216 ( 0.2092) | 0.0900 ( 0.2407) |
| 0.1563 | 0.371 | 0.3732 ( -0.0022) | 0.3093 ( 0.0617) | 0.3624 ( 0.0085) | 0.2648 ( 0.1061) | 0.3224 ( 0.0485) | 0.2035 ( 0.1675) | 0.2288 ( 0.1422) | 0.1407 ( 0.2302) | 0.1182 ( 0.2528) | 0.0913 ( 0.2797) |
| 0.0938 | 0.3263 | 0.3289 ( -0.0027) | 0.2749 ( 0.0513) | 0.3177 ( 0.0086) | 0.2367 ( 0.0896) | 0.2783 ( 0.0480) | 0.1824 ( 0.1439) | 0.1938 ( 0.1325) | 0.1255 ( 0.2008) | 0.0992 ( 0.2270) | 0.0787 ( 0.2475) |
| 0.0781 | 0.3035 | 0.3061 ( -0.0025) | 0.2578 ( 0.0457) | 0.2950 ( 0.0086) | 0.2221 ( 0.0815) | 0.2575 ( 0.0461) | 0.1709 ( 0.1327) | 0.1814 ( 0.1221) | 0.1197 ( 0.1838) | 0.0940 ( 0.2095) | 0.0752 ( 0.2283) |
| 0.0703 | 0.2901 | 0.2925 ( -0.0024) | 0.2470 ( 0.0431) | 0.2820 ( 0.0081) | 0.2132 ( 0.0769) | 0.2459 ( 0.0442) | 0.1640 ( 0.1261) | 0.1717 ( 0.1184) | 0.1136 ( 0.1765) | 0.0914 ( 0.1987) | 0.0734 ( 0.2167) |
| 0.0625 | 0.2748 | 0.2771 ( -0.0022) | 0.2345 ( 0.0404) | 0.2667 ( 0.0082) | 0.2020 ( 0.0728) | 0.2343 ( 0.0405) | 0.1571 ( 0.1177) | 0.1595 ( 0.1153) | 0.1054 ( 0.1695) | 0.0889 ( 0.1860) | 0.0717 ( 0.2032) |
| 0.0000 | 0.0000 | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) |

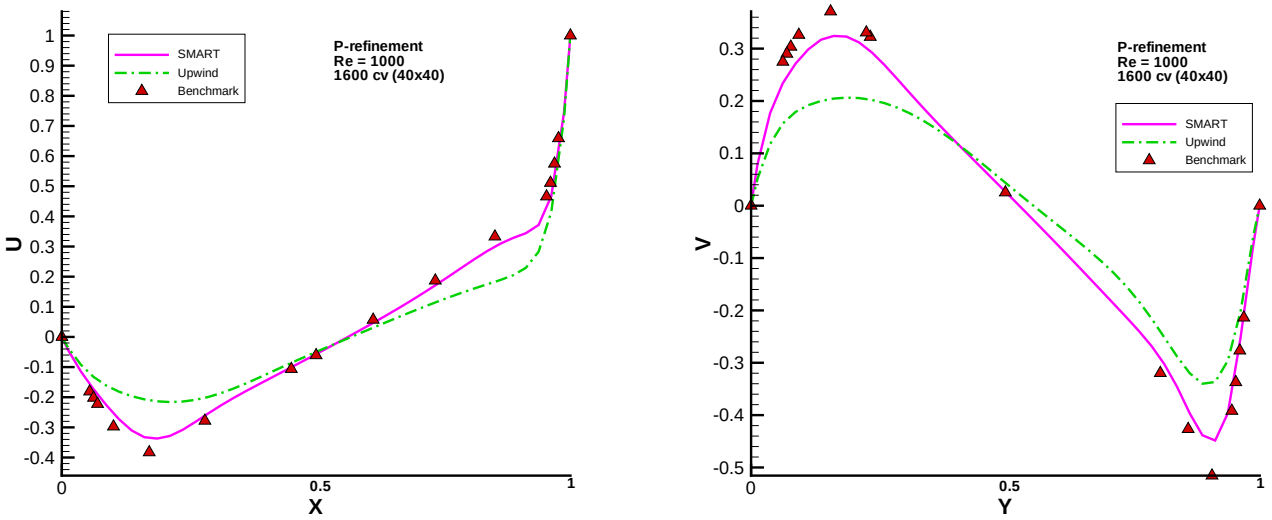


Figure 6: Comparison of SMART and upwind schemes on mesh with 1600cv (40x40) .

Comments:
Results of SMART scheme for this problem are obviously much more accurate than results of upwind scheme.

## 2.2 Differential-Heated Cavity

Boundary conditions:

$u = 0,\ v = 0,\ \dfrac{\partial T}{\partial y} = 0,\quad y = 0,\ 0 \le x \le L_x;\qquad u = u_0,\ v = 0, \dfrac{\partial T}{\partial y} = 0,\quad y = L_y,\ 0 \le x \le L_x;$

$u = 0,\ v = 0,\ T = T_1,\quad x = 0,\ 0 \le y \le L_y;\qquad u = 0,\ v = 0,\ T = T_2,\quad x = L_x,\ 0 \le y \le L_y.$

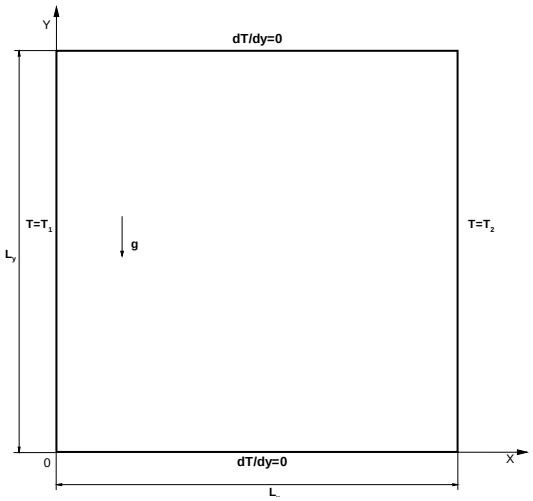


Figure 7: Differential-Heated Cavity problem scketch

## 2.3 Numerical results

Caracteristics of the solution:

$$x^* = \frac{x}{L},\quad y^* = \frac{y}{L},\quad u^* = \frac{uL\rho c_P}{\lambda},\quad v^* = \frac{vL\rho c_p}{\lambda},\quad Nu_y = \frac{L(T_y - T_1)}{\triangle x(T_2 - T_1)}$$

**Ra** $= 10^5$**, Pr=0.71**

Problem parameters:

$L_x = L_y = 1,\ \rho = 10^{-4},$ ‘$\mu = 1.704 * 10^{-5},\ \beta = 4.0896 * 10^{-4},$

$\lambda = 0.024,\ c_p = 10^3,\ g = 10^5,\ T_1 = 100,\ T_2 = 0,\ T_\infty = 50.$

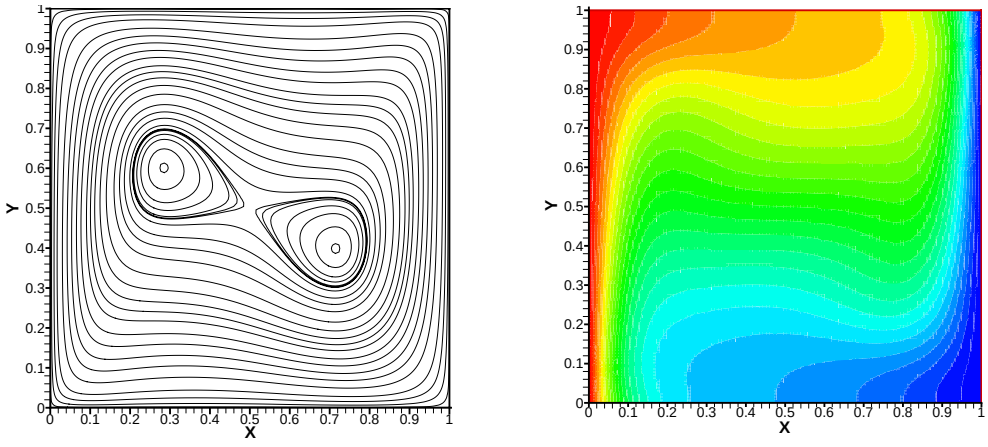


Figure 8: Ra=$10^5$. Streamline contours (left) and isotherms (right). Mesh 25600cv, SMART scheme.

Table 4: Comparison of SMART and PowerLaw schemes for Ra=$10^5$

| Mesh | Benchmark | 100cv (10x10) SMART | 100cv (10x10) PDS | 400cv (20x20) SMART | 400cv (20x20) PDS | 1600cv (40x40) SMART | 1600cv (40x40) PDS | 6400cv (80x80) SMART | 6400cv (80x80) PDS | 25600cv (160x160) SMART | 25600cv (160x160) PDS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $u^*_{max}$ | 34.730 | 37.645 (8.393) | 34.086 (-1.85 ) | 34.717 (-0.03 ) | 34.843 (0.32 ) | 34.607 (-0.35) | 34.761 (0.06) | 34.708 (-0.06) | 34.744 (0.04) | 34.733 (0.01) | 34.741 (0.03) |
| $y^*$ | 0.855 | 0.824 (-6.89) | 0.824 (-6.89) | 0.865 (1.16) | 0.865 (1.16) | 0.856 (0.11) | 0.856 (0.11) | 0.854 (-0.11) | 0.854 (-0.11) | 0.856 (0.11) | 0.856 (0.11) |
| $v^*_{max}$ | 68.59 | 68.6649 (-3.62) | 65.359 (-4.71) | 67.859 (-1.06) | 68.010 (-0.84) | 68.723 (0.19) | 68.735 (0.21) | 68.644 (0.07) | 68.628 (0.05) | 68.650 (0.08) | 68.643 (0.07) |
| $x^*$ | 0.066 | 0.0833 (26.21) | 0.0833 (26.21) | 0.075 (13.63) | 0.075 (13.63) | 0.0675 (2.27) | 0.0675 (2.27) | 0.0637 (-3.48) | 0.0637 (-3.48) | 0.0656 (-0.6) | 0.0656 (-0.6) |
| $Nu_{max}$ | 7.717 | 9.303 (20.55) | 9.330 (20.90) | 8.762 (13.54) | 8.376 (8.53) | 8.08 (4.7) | 7.893 (2.28) | 7.799 (1.06) | 7.764 (0.60) | 7.736 ( 0.24) | 7.731 (0.18) |
| $y^*$ | 0.081 | 0.0833 (2.83) | 0.0833 (2.83) | 0.075 (-7.40) | 0.075 (-7.40) | 0.0675 (1.85) | 0.0825 (1.85) | 0.079 (-2.46) | 0.078 (-3.7) | 0.0806 (-0.49) | 0.0806 (-0.49) |
| $Nu_{min}$ | 0.729 | 1.406 (92.86) | 0.801 (9.87) | 0.870 (19.34) | 0 .777 (9.87) | 0.746 (2.33) | 0.744 (1.92) | 0.730 (0.13) | 0.732 (0.41) | 0.728 (-0.13) | 0.7292 (0.02) |
| $y^*$ | 1.0 | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 ( 0 ) | 1.0 (0) | 0.998 (-0.19) |
| $Nu_{mig}$ | 4.519 | 5.4198 (19.93) | 4.854 (7.41) | 4.841 (7.12) | 4.638 (2.63) | 4.606 (1.92) | 4.549 (0.66) | 4.539 (0.44) | 4.528 (0.19) | 4.525 (0.13) | 4.523 (0.08) |

Comments:
SMART scheme appeared to be not better than of PDS scheme. This schemes give different errors and I can't say wich scheme is better for this problem. On the mesh with 25600cv SMART and PDS give almost equal result on 5 characteristic values, SMART gives better result on 2 characteristic values and PDS gives better result on 2 charecteristic values. Probably SMART is better than PDS on finer meshes, because it is clear that SMART scheme gives less accurate result on coarser meshes.